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Identifying similarity and anomalies for cryptocurrency moments and distribution extremities

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Abstract

We propose two new methods for identifying similarity and anomalies among collections of time series, and apply these methods to analyse cryptocurrencies. First, we analyse change points with respect to various distribution moments, considering these points as signals of erratic behaviour and potential risk. This technique uses the MJ_1 semi-metric, from the more general MJ_p class of semi-metrics [James *et al.*, 2019], to measure distance between these change point sets. Prior work on this topic fails to consider data between change points, and in particular, does not justify the utility of this change point analysis. Therefore, we introduce a second method to determine similarity between time series, in this instance with respect to their extreme values, or tail behaviour. Finally, we measure the consistency between our two methods, that is, structural break versus tail behaviour similarity. With cryptocurrency investment as an apt example of erratic, extreme behaviour, we notice an impressive consistency between these two methods.

1 Introduction

The cryptocurrency market is in its relative infancy, and is characterised by large price fluctuations, significant volatility, and a high degree of similarity in movement. Given cryptocurrencies' technological grounding, it has been of great interest to the field of computer science and quite naturally, machine learning researchers have been eager to conduct research in the field. Large investment groups tend to dominate prominent and highly liquid exchange-traded financial products such as equity markets, fixed income markets, equity and fixed income indices, many derivative markets etc. These investment groups often marry their investment processes with underlying econometric and financial theory, enforcing efficient markets-type thinking. There has been reluctance from many large and established investment managers to invest in cryptocurrencies. Accordingly, retail investors and less procedural investors make up a larger proportion of the cryptocurrency market. The cryptocurrency market is possibly the best representation of crowd behaviour and the associated chaos within financial markets.

There is significant literature studying the variance of cryptocurrencies. Most research has focused on applying ARCH and GARCH-style models, stochastic volatility and CARR models to model the volatility of individual cryptocurrencies, such as in [Katsiampa, 2017; Chu *et al.*, 2017] and others. Despite the highly non-stationary behaviour that cryptocurrencies often exhibit, these models do not segment the time series into locally stationary segments. While [Phillip *et al.*, 2019] apply a buffer threshold model with different forward and backward change points, they consider one cryptocurrency at a time rather than changes across cryptocurrencies.

[Hawkins, 1977], and later [Hawkins *et al.*, 2003; Hawkins and Zamba, 2005; Hawkins and Deng, 2010] introduced change point models, a developing field of algorithms designed to break time series into locally stationary segments, where each segment is governed by a separate probability distribution function. Since then, the work has been extended by several authors, most significantly Ross et al. [Ross and Adams, 2011, 2012; Ross *et al.*, 2011]. These methods were importantly consolidated by [Ross, 2015], where software was designed to easily apply these change point algorithms to various time series. In this paper, we apply the Mann-Whitney test [Ross, 2015; Pettitt, 1979] as one of the methods to detect change points in cryptocurrency log returns and variance.

[James *et al.*, 2019] apply semi-metrics, a recent development of the field of metric spaces, to evaluate distances between sets of change points and determine similarity in structural breaks for a collection of time series. Metric spaces are fundamental in mathematics, and have been used to measure distances between finite sets of points for several years. The most commonly used of these is the Hausdorff metric, used by [Atallah *et al.*, 1991; Barton *et al.*, 2010] and others. The Hausdorff metric has a flaw - its sensitivity to outliers - discussed in [Baddeley, 1992]. Improvements in this sensitivity can be provided by the use of semi-metrics, which sacrifice the triangle inequality property of a metric. Semi-metrics have been used in various applications where a distance is needed, such as [Jacobs *et al.*, 2000] and [Rucklidge, 1996].

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This paper proposes two new approaches to the cryptocurrency literature. We use first the MJ_p family of semi-metrics and second a modified Wasserstein metric procedure, rather than classical time series models, to uncover patterns within large collections of time series. The methods we apply are flexible, and may build upon any underlying criteria for change point identification. There is also flexibility to use other distances explored in [James *et al.*, 2019] such as the $\mathrm{MJ}_{0.5}$ and MJ_2 distances, or to modify the proportion of the tail we extract for the second method.

2 An algorithm for similarity and anomaly identification

2.1 Generating the distance matrix

Given a collection of time series indexed by days, we apply the two phase change point detection algorithm detailed by [Ross, 2015] to generate sets of change points. This algorithm may be modified by choosing different threshold levels appropriate for the context. Then, we apply the MJ_1 distance of [James *et al.*, 2019] between sets of change points. This distance calculates the L^1 norm average of all the minimum distances from a set S to a set T and from T to S, that is,

$$D_1(S,T) = \frac{1}{2}\left(\frac{\sum_{t\in T} d(t,S)}{|T|} + \frac{\sum_{s\in S} d(s,T)}{|S|}\right) \tag{1}$$

where $d(t,S)$ is the distance from a point t to a set S.

Given n time series, extract their change point sets $S_1, ..., S_n$. Then form the distance matrix $D_{ij} = D_1(S_i, S_j), i, j = 1, ..., n$.

2.2 Analysing the distance matrix

We propose three ways of analysing this matrix of distances between time series. First, one can extract, order and plot the absolute values of the eigenvalues $|\lambda_1| < ... < |\lambda_n|$. For both the log returns and variance time series, if many eigenvalues are relatively close to zero, relative to normalisation by the length of the total time period, one may conclude that correspondingly many of the time series are highly similar.

The second approach is hierarchical clustering on the distance matrix, which allows us to see which time series are most similar based on the MJ_1 semi-metric. This method produces easy to interpret dendrograms, seen in Figures 7c and 8c below.

The third approach uses spectral clustering on the graph Laplacian matrix to aid in detecting groups of similarity, as well as anomalies. The distance matrix D is transformed into an affinity matrix A as follows:

$$A_{ij} = 1 - \frac{D_{ij}}{\max_{k,l}\{D_{kl}\}} \tag{2}$$

The graph Laplacian matrix is given by:

$$L = E - A, \tag{3}$$

where E is the diagonal degree matrix with $E_{ii} = \sum_j A_{ij}$.

The eigenvectors of the graph Laplacian are then clustered using a standard algorithm such as K-means.

Transitivity preservation

Unlike metrics, semi-metrics may not satisfy the triangle inequality. This presents a theoretical problem: $d(S,T)$ and $d(T,R)$ could both be small, while $d(S,R)$ could be large. Under the semi-metric, S would then be close to T, and T close to R, but S not close to R; transitivity of proximity may not be preserved and our semi-metric would not be an appropriate distance measure. To address this concern, we generate a three dimensional matrix to test whether the triangle inequality is satisfied for all possible triples, and to show that our measure is reliably transitive. The matrix is coloured as follows:

$$T_{i,j,k} = \begin{cases} \text{blue}, & \frac{D(i,k)}{D(i,j)+D(j,k)} \leq 1 \\ \text{yellow}, & 1 < \frac{D(i,k)}{D(i,j)+D(j,k)} \leq 2 \\ \text{red}, & \text{else} \end{cases} \tag{4}$$

If T is overwhelmingly blue and yellow, our metric is violating the triangle inequality infrequently and mildly, so that it still respects the transitivity property of proximity.

3 Modelling distributional extremities

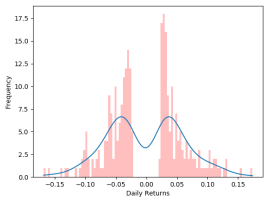
In this section we seek to do the following:

1. Highlight similarity and anomalies between cryptocurrencies with respect to their tail behaviour.
2. Restrict our attention to the top and bottom 5% of the log returns, and examine the associated restricted distribution. Cryptocurrencies may have different behaviour in extreme circumstances.
3. Compute distance between these restricted distributions using the Wasserstein distance. These distances are stored in a new distance matrix. We show that Tether is anomalous with respect to tail behaviour.
4. Examine differences between the change point and tail distance matrices. Hence, determine cryptocurrencies which are anomalous with respect to the consistency between structural breaks and tail behaviour. That is, investigate which, if any, cryptocurrencies are distinguishing themselves from the rest of the collection in only one of change point behaviour, or tail behaviour, but not both,
5. Thus, develop a method which determines changes in structural breaks in mean, variance, etc, but that can also differentiate extreme behaviours, akin to characteristics such as skew and kurtosis. Finally, compare anomalies with respect to structural breaks and tail distribution with a so-called consistency matrix.

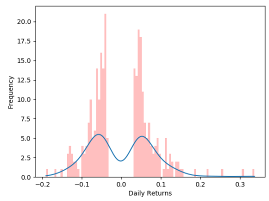
3.1 Generating bi-modal tail distribution

In this section we describe the mathematical procedure outlined above. Begin with a continuous probability measure $\mu = f(x)dx$, where dx is Lebesgue measure, and $f(x)$ a probability density function. As such, $f(x)$ is non-negative everywhere with integral 1. We first extract the points of density 5% and 95% respectively by the equations

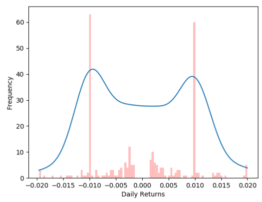
$$\int_{-\infty}^{s} f(x)dx = 0.05 \tag{5}$$



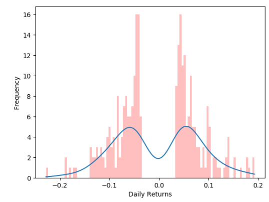
(a) Bitcoin tail distribution



(b) Litecoin tail distribution



(c) Tether tail distribution



(d) Monero tail distribution

Figure 1: Distribution of extreme values for cryptocurrency log returns

$$\int_{t}^{\infty} f(x)dx = 0.05 \tag{6}$$

The left tail is then given by $x \leq s$, while the right is given by $x \geq t$. Next form the restricted function by

$$g(x) = f(x)\mathbb{1}_{\{x\leq s\}\cup\{x\geq t\}} = \begin{cases} f(x), x \leq s \\ 0, s < x < t \\ f(x), x \geq t \end{cases}$$

Above, $\mathbb{1}$ denotes an indicator function of a set; this construction essentially truncates f only in its tail range.

Next, we form the associated Radon-Nikodym measure $\nu = g(x)dx$ where dx is Lebesgue measure.

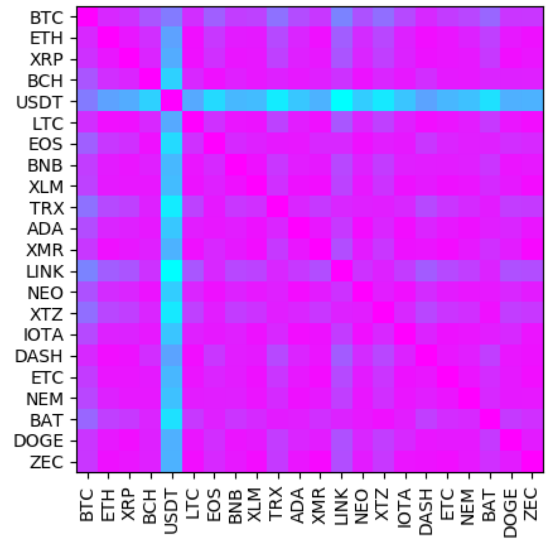
Now suppose we are given n probability measures associated to n cryptocurrencies, $\mu_1, ..., \mu_n$. Form the corresponding restricted measures $\nu_1, ..., \nu_n$, compute the Wasserstein distances $d^W(\nu_i, \nu_j)$ between them, and record these in a matrix $D^W_{ij} = d^w(\nu_i, \nu_j)$.

This procedure works even more simply for a discrete distribution, given by a finite data set, for instance. One simply forms the empirical distribution function, then removes the middle 90% of the values by order.

Finally, for representational convenience we include the kernel destiny estimation plots in Figure 1.

3.2 Distance between extreme behaviours

In Figure 2a, we depict on a dendrogram these Wasserstein distances between the restricted measures. This captures the similarity between the tail behaviour (top and bottom 5%) of these cryptocurrencies. We can see that Tether has a very different tail distribution. This is confirmed by Figure 1c above. The distribution is much more narrow, seen by limited range on the x-axis compared to Figures 1a, 1b and 1d. We are likely detecting significantly less kurtosis.



(a) Distance between distributional extremities

Figure 2: Wasserstein distance between tail behaviour distributions

3.3 Consistency between change point and tail distances

In [James *et al.*, 2019], James et al. imply, but do not explicitly discuss, that change points may constitute a representation of extreme behaviour. After all, a random variable or real world quantity changing its statistical properties represents an erratic event. Extending this idea, we explicitly examine the connection and consistency between structural break and tail behaviour distance analysis. Using semi-metrics or metrics to differentiate cryptocurrencies based on these two attributes serves a similar purpose: to interpret similarity and anomalies based on extreme or erratic behaviour.

To perform this analysis, we form both distance matrices D^W from section 3.1 and D^{MJ} from 2.1, and transform each into an affinity matrix A^W and A^{MJ} according to the procedure in equation 2. All elements of these affinity matrices lie in $[0, 1]$ so it is appropriate to compare them directly by taking their difference

$$C = A^{MJ} - A^W \tag{7}$$

Term this the *consistency matrix* between structural breaks and tail behaviour. A heat map of this matrix is depicted in Figure 4.

Finally, we perform hierarchical clustering on this consistency matrix to determine if there are any anomalies. Results are displayed in Figure 6a.

4 Experiments and results

We run two cryptocurrency experiments, analysing log returns and variance for 22 cryptocurrencies. In each experiment, the Mann-Whitney test is applied to identify the number and

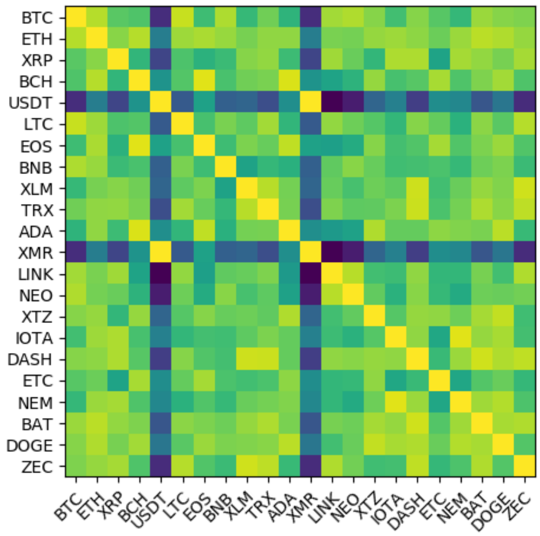
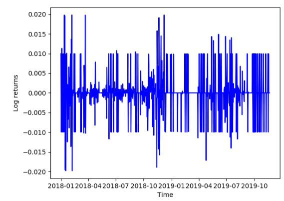
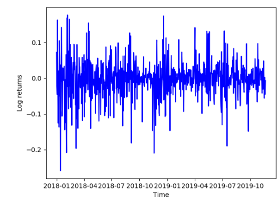


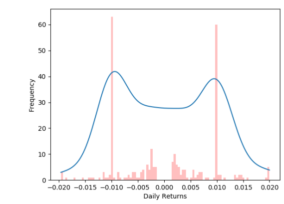
Figure 3: MJ_1 affinity matrix



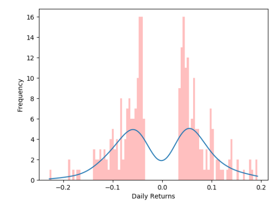
(a) Tether log returns vs time



(b) Monero log returns vs time



(c) Tether tail distribution



(d) Monero tail distribution

Figure 5: Tether log returns, Monero log returns, Tether distribution, Monero Distribution

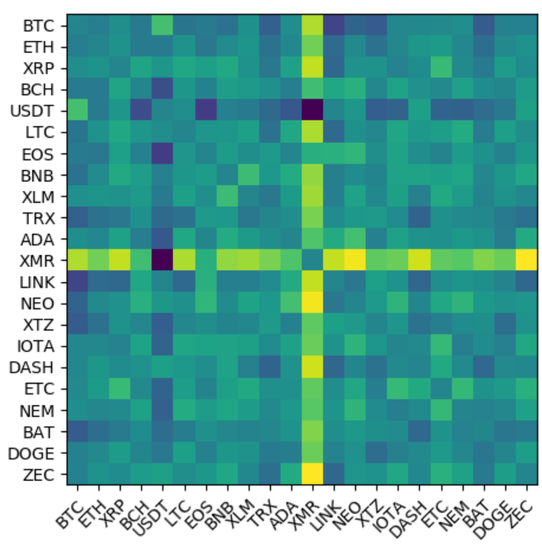
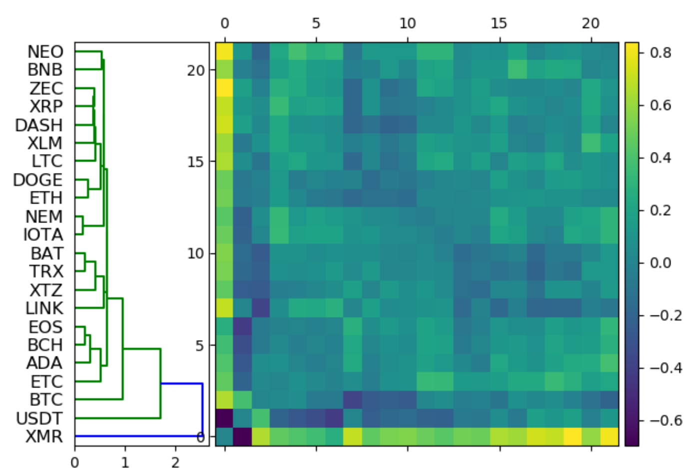


Figure 4: Consistency between structural breaks and tail behaviour



(a) Consistency between structural breaks and tail behaviour

Figure 6: Hierarchical clustering on difference between affinity matrices- consistency matrix - for distance between structural breaks and distance between tail distributions

locations of change points, and the respective distance matrix is analysed with our three methods.

The data we analyse is taken from Coinmarketcap. Among the 30 largest cryptocurrencies by market capitalisation, we include only those with price histories which go as far back as 01-01-2018. Twenty-two cryptocurrencies remain in our collection. We analyse the daily log returns and Parkinson range variance measures of these cryptocurrencies between 01-01-2018 and 19-11-2019, a period of 688 days. Log returns are calculated by $R_t = \log(\frac{P_t}{P_{t-1}})$ and the Parkinson range variance is calculated as, $\sigma_t^2 = \frac{(\log H_t - \log L_t)^2}{4 \log 2}$. Finally, we extract the bi-modal tail distributions according to the procedure of section 3.1, and analyse the consistency with change points as described in section 3.3.

Similarity in log returns structural breaks

The first experiment determines the similarity between cryptocurrencies with respect to change points in the log returns series. Figure 7a demonstrates there is a high degree of similarity in the returns of the cryptocurrency market. Approximately 16 eigenvalues are less than 200 in absolute value, small relative to the total time period of 688 days. It is clear from this plot that there are at least two outliers, with eigenvalues well outside the typical range for the time series collection.

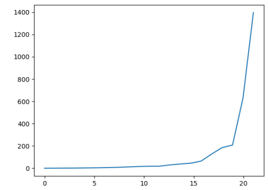
The distance matrix's dendrogram in Figure 7c demonstrates that there is one cluster of broad similarity, and several outliers, including Tether and Monero. Within the large cluster of similar cryptocurrencies, there are sub-clusters of highly similar cryptocurrencies. There are approximately 4 sub-clusters of extreme similarity around the matrix diagonal. Both Tether and Monero are identified as dissimilar to most cryptocurrencies, however they are determined to be similar to each other. In fact, the truth is even starker than the dendrogram reveals: both Tether and Monero have an empty set of change points, and hence the MJ_1 distance between these sets is trivially 0. An empty set of change points means their log returns are determined by the algorithm to be governed by the same probability distribution for the entire period. Indeed, one can see this in Figures 5a and 5b. The lack of any change points is strikingly different to the rest of the collection. This is an important insight, highlighting that there may be similarity among cryptocurrencies anomalous from the majority of the collection.

The third method, spectral clustering, algorithmically agrees with hierarchical clustering, and shows that there are two clusters within the data, one containing Tether and Monero, and one containing all others.

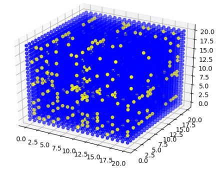
Similarity in variance structural breaks

The second experiment determines the similarity between cryptocurrencies with respect to change points in the time series' variance.

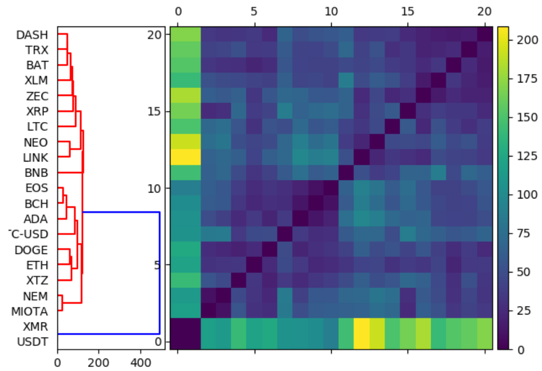
An analysis of the distance matrix indicates more inherent similarity with respect to structural breaks in the variance than for the log returns. Figure 8a shows that all eigenvalues are less than 160, even smaller with respect to the time period of 688 days; in Figure 7a, one eigenvalue was as high as 1400. The dendrogram in Figure 8c has two key insights.



(a) Eigenvalue plot

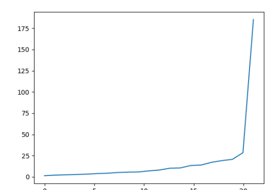


(b) Transitivity plot

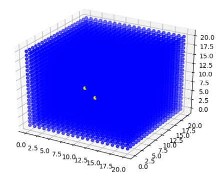


(c) Dendrogram plot

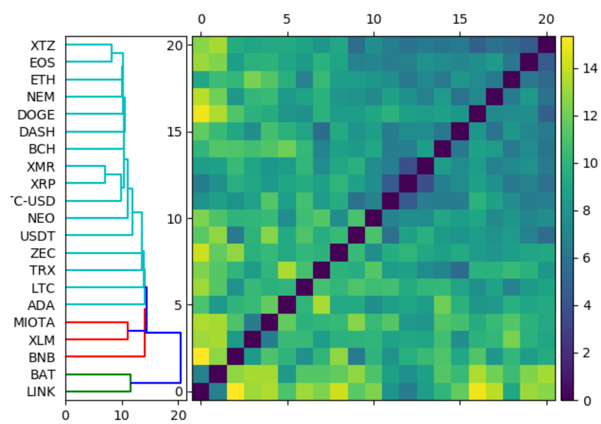
Figure 7: Results of distance matrix analyses for change points identified using Mann-Whitney test applied to cryptocurrency log returns



(a) Eigenvalue plot



(b) Transitivity plot



(c) Dendrogram plot

Figure 8: Results of distance matrix analyses for change points identified using Mann-Whitney test applied to cryptocurrency variance

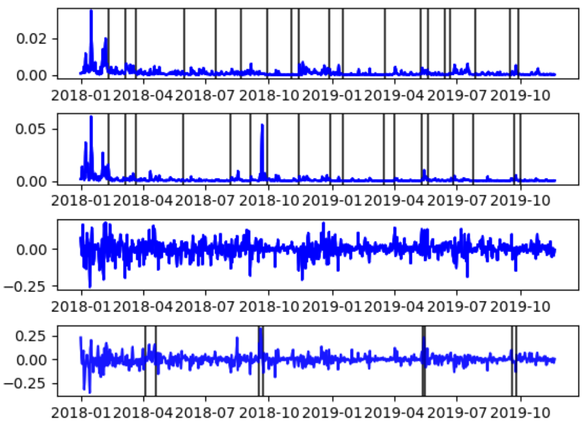


Figure 9: Log returns and Parkinson daily variance for Monero and Ripple. x axis represents time. Plots 1 and 2 display Parkinson daily variance measure with change points for Monero and Ripple; plots 3 and 4 display log returns with change points for Monero and Ripple. The structural breaks are more frequent, and more similar in the variance time series.

First, the scale of the dendrogram in Figure 8c is 20 times smaller than that of Figure 7c, suggesting that anomalies are far less significant when determining structural breaks in variance. Second, although the general degree of similarity is much higher between change points in the variance, the pattern of highly similar clusters of cryptocurrencies grouping around the diagonal has disappeared. Given the 20 fold difference in scale, the proper interpretation of this is that all 22 cryptocurrencies form one cluster. Hierarchical clustering indicates essentially all 22 cryptocurrencies have very similar structural breaks in variance. Spectral clustering indicates that there are three clusters of cryptocurrencies. One cluster contains Chainlink, another contains Basic Attention and the third cluster contains the remainder of the cryptocurrencies.

In Figure 9, we illustrate the distinct differences in similarity between the structural breaks of log returns and variance between two cryptocurrencies, Monero and Ripple. These were identified as highly similar with respect to variance, but reasonably dissimilar in log returns. Plots 1 and 2 in Figure 9 display a high degree of similarity in the structural breaks corresponding to variance. On the other hand, plots 3 and 4 in Figure 9 show that the log returns display significantly less similar patterns with respect to structural breaks. These results further confirm the ability of the semi-metric MJ_1 to identify anomalous patterns within the cryptocurrency market. This could provide insights for trading ideas and decision support in asset allocation decisions for portfolios of cryptocurrencies and multi-asset portfolios that contain cryptocurrencies. This analysis also highlights that the cryptocurrency market may provide exposure to highly correlated investment risk, as the variance of most cryptocurrencies exhibits pronounced synchronicity.

Consistency between structural break and tail distribution analysis

In this section, we compare and contrast the structural breaks and tail distributions of the log returns of our collection of cryptocurrencies. We form the consistency matrix described in section 3.3, depicted in Figure 4 and analyse the hierarchical clustering in Figure 6a.

In interpreting these, pay close attention to both Tether and Monero. Recall each of these cryptocurrencies is determined by our algorithm to have an empty set of change points. One can see this regularity of returns with time in Figures 5a and 5b. As such, their MJ distance between their change points is 0, and they form their own anomalous cluster in Figure 7c. That is, in change point analysis, Tether and Monero are the two anomalies. On the other hand, with respect to tail distributions, Tether is the only anomalous cryptocurrency, as seen in Figure 1c. Monero is rather similar to other cryptocurrencies in terms of tail distribution.

And fittingly, it is only Monero which is identified as anomalous in terms of inconsistencies between tail behaviour and structural breaks. That is, it is the unique cryptocurrency for which the tail distribution and structural break analysis are yielding different results. This supports our interest in the distance between change point sets as a measure of distance between random variables based on their extreme and erratic behaviour.

5 Conclusion

We use the MJ_1 semi-metric to measure similarity between 22 cryptocurrencies, according to the change points of log returns and variance. We also compare the consistency between our change point analysis and a tail distribution analysis.

Our analysis of distance matrices suggests that the cryptocurrency market is highly similar with respect to structural breaks in both log returns and variance, but anomalous behaviour is more profound in log returns than variance. Subclustering according to change points of cryptocurrency returns suggests that members within each sub-cluster should not appear together in a portfolio to minimise risk. Identifying these clusters may also provide investment opportunities within the cryptocurrency market; in particular, analysing clusters holistically may provide opportunities for pairs trading. On the other hand, change points of cryptocurrency variance form essentially one cluster that covers the entire market. The extraordinary similarity of the variance among the cryptocurrency market agrees with previous findings that the cryptocurrency market is highly volatile and risky.

Reduction of such extraordinary volatility is among the highest priorities for investors in the cryptocurrency market. We have analysed change points, which can herald erratic behaviour, and the tail end distributions, which control extreme behaviour. Our two methods to monitor similarity and anomalies may provide new insights in the reduction of risk. Their significant consistency means they will not provide contradictory recommendations, and together they could offer unique insights that one method alone could miss.

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